behaviors of the CC 31031 prototype and that obtained according to [2].

IV. Conclusions

A broad-band equivalent circuit of a generic microwave planar network has been derived in terms of lumped-constant elements. These elements are only smoothly frequency dependent, because of the dispersion properties of microstrips, so that they may be considered, with good approximation, to be constant with the frequency, even in broad-band simulations.

Contrary to previously proposed equivalent circuits, which are strongly frequency dependent, the present one is easy to handle and can be a useful basis for designing microstrip planar structures starting from conventional synthesis procedures.

Experiments performed up to 12.5 GHz on structures with different geometries have shown good agreement with theoretical results.

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High-Accuracy Numerical Data on Propagation Characteristics of α-Power Graded-Core Fibers

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***Abstract*—High-accuracy data of normalized cutoff frequencies, propagation constants, and delay time of LP_{ml} modes for α-power graded-core fibers ($\alpha = 1$, 2, 4, and 10) are obtained by using two entirely different methods: power-series expansion and finite element methods, and the results are compared. The difference between cutoff frequencies obtained by these methods is less than 0.005 percent for most of the LP modes. The obtained data are accurate enough to be used as the standard for estimating the accuracy of other various analyses.**

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I. Introduction

VARIOUS methods have been presented for the analysis of propagation characteristics of optical fibers having arbitrary refractive-index profiles. Examples of those are the WKB method, [1] power-series expansion method, [2] Rayleigh–Ritz method [3], finite element

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method [4], and staircase-approximation method [5].

However, the accuracy of these analyses have not been investigated systematically. One of the reasons is that the high-accuracy, reliable "standard" data on the propagation characteristics for typical graded profiles, which are necessary for estimating the accuracy, are not available. (Accurate data for uniform-core fibers can be obtained easily by an analytical approach [6].)

The major purpose of this paper is to present such high-accuracy standard data within scalar-wave approximation. Normalized cutoff frequencies, propagation constants, and delay times are shown for α-power profiles where $\alpha=1$, 2, 4, and 10. To assure accuracy, two entirely different methods, power-series expansion and finite-element methods, are used and the results are compared.

The second purpose is to show the formulation for the delay-time calculation by the power-series expansion method. Gambling *et al.* derived the dispersion equation for α-power single-mode fibers for integral values of α [2]. Afterwards Love extended that dispersion equation to the cases of higher modes and arbitrary rational values of α [7]. However, the equation for the delay-time calculation has not been presented. In this paper the delay-time equation is derived from the variational expression of the propagation constant.

II. α-Power Profiles

We consider α-power refractive-index profiles expressed as

$$n(r)=n_1\left[1-2\rho\Delta(r/a)^{\alpha}\right]^{1/2}, \qquad 0\leqslant r\leqslant a \tag{1a}$$

$$=n_2=n_1\left[1-2\Delta\right]^{1/2}, \qquad a<r \tag{1b}$$

where a denotes the core radius, n_1 and n_2 are the refractive indices upon the axis and in the cladding, respectively, Δ is the relative refractive-index difference between the core axis and cladding ($\Delta=(n_1^2-n_2^2)/2n_1^2$), and ρ is a parameter representing the refractive-index step or valley at the core-cladding boundary. A smooth continuation at the core-cladding boundary, the presence of a step, and that of a valley are expressed by $\rho=1$, $\rho<1$, and $\rho>1$, respectively.

III. Power-Series Expansion Method (PSEM)

In this method (PSEM), both the refractive-index profile and the field-distribution function are expanded in power series, and these are put into a wave equation to determine the coefficients of the series for the field function.

A. Dispersion Equation

The scalar wave equation can be written as [8]

$$\frac{1}{x}\frac{d}{dx}\left(x\frac{dR}{dx}\right)+\left(u^2-\rho v^2x^{\alpha}-\frac{m^2}{x^2}\right)R=0 \tag{2}$$

where

$$v^2=a^2k^2n_1^2(2\Delta): \text{normalized frequency} \tag{3}$$

$$u^2=a^2(n_1^2k^2-\beta^2) \tag{4}$$

$$x=r/a \tag{5}$$

$R(x)$ is the function representing the field distribution, m is the rotational mode number, and k and β denote the propagation constants in free space and in the fiber along its axis, respectively.

We express the solution in the core as [2]

$$R_{\text{core}}(x)=Ax^m\sum_{n=0}^{\infty}a_nx^n, \qquad x\leqslant 1 \tag{6}$$

and that in the cladding as

$$R_{\text{cladding}}(x)=BK_m(wx), \qquad x>1 \tag{7}$$

where A and B are constants, K_m is the mth order modified Bessel function of the second kind, and

$$w^2=v^2-u^2=a^2(\beta^2-k^2n_2^2). \tag{8}$$

The coefficients a_n in (6) are determined by the following recurrence formula [7]:

$$a_n=\begin{cases}-\dfrac{-1}{n(n+2m)}u^2a_{n-2}, & 2\leqslant n<\alpha+2 \quad (9a)\\ -\dfrac{-1}{n(n+2m)}\left(u^2a_{n-2}-\rho v^2a_{n-\alpha-2}\right), & \alpha+2\leqslant n \quad (9b)\end{cases}$$

$$a_0=1 \tag{10}$$

$$a_1=0. \tag{11}$$

From the continuity of R and dR/dx at $x=1$, we obtain the dispersion equation as [7]

$$\frac{\sum_{n=0}^{\infty}na_n}{\sum_{n=0}^{\infty}a_n}+2m+\frac{wK_{m-1}(w)}{K_m(w)}=0. \tag{12}$$

B. Cutoff Conditions

At cutoff frequencies for each mode

$$u=v \tag{13}$$

$$w=0. \tag{14}$$

Hence (12) becomes

$$\frac{\sum_{n=0}^{\infty}nb_n}{\sum_{n=0}^{\infty}b_n}+2m=0 \tag{15}$$

where

$$b_0=1 \tag{16}$$

$$b_1=0 \tag{17}$$

$$b_n=\begin{cases}\dfrac{-1}{n(n+2m)}v^2b_{n-2}, & 2\leqslant n<\alpha+2 \quad (18a)\\ \dfrac{-1}{n(n+2m)}v^2\left(b_{n-2}-\rho b_{n-\alpha-2}\right), & \alpha+2\leqslant n. \quad (18b)\end{cases}$$

The normalized cutoff frequencies v_c can be determined by solving (15)–(18). The lth smallest solution of v gives the v_c for LP_{ml} mode.

C. Delay Time

When the dispersion relation ($k-\beta$ relation) is obtained in a form of $f(k,\beta)=0$, the delay time per unit distance is given as

$$t=-\frac{1}{c}\frac{\partial f/\partial k}{\partial f/\partial \beta} \tag{19}$$

where c is the velocity of light. Using (12), we obtain

$$t=-\frac{1}{c}\frac{\frac{\partial}{\partial k}\left[\sum_{n=0}^{\infty} na_n/\sum_{n=0}^{\infty} a_n+wK_{m-1}(w)/K_m(w)\right]}{\frac{\partial}{\partial \beta}\left[\sum_{n=0}^{\infty} na_n/\sum_{n=0}^{\infty} a_n+wK_{m-1}(w)/K_m(w)\right]}. \tag{20}$$

However, computation of the delay time using this formula takes a long computer time to assure accuracy. A better expression is obtained from the variational expression of β [3] as

$$t=\frac{\int_0^{t}\frac{d(k^2n^2(x))}{dk}R^2(x)x\,dx}{c\beta\int_0^{\infty}R^2(x)x\,dx}. \tag{21}$$

Putting (6) and (7) into (21), we obtain the delay time as

$$t=\frac{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_ia_j/i+j+\alpha+2m+2+\left(\sum_{i=0}^{\infty} a_i\right)^2\eta_m}{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_ia_j/i+j+2m+2+\left(\sum_{i=0}^{\infty} a_i\right)^2\eta_m} \tag{22}$$

where

$$\eta_m=\frac{K_{m-1}(w)K_{m+1}(w)}{K_m^2(w)}-1. \tag{23}$$

IV. Finite Element Method (FEM)

In this method (hereafter FEM), the wave equation is translated into a corresponding variational problem, which is then solved by the FEM approach to obtain the proper equation. The formulation was first described in [4] for both the vectorial and scalar wave analyses, and later in [9] in a much simpler form applicable only to scalar wave analysis.

The process of the computation using the FEM is omitted in this paper because it is essentially identical to one described in [9].

V. Results of Numerical Analyses

A. Normalized Cutoff Frequencies for Uniform-Core Fibers

First, to estimate the accuracy of the PSEM and FEM independently, the normalized cutoff frequencies of LP_{1l} modes in a uniform-core fiber calculated by using each method are compared with exact values. For a uniform core fiber, the exact values are given analytically as lth roots of $J_1(x)=0$, where $J_1(x)$ denotes the first order Bessel function of the first kind.

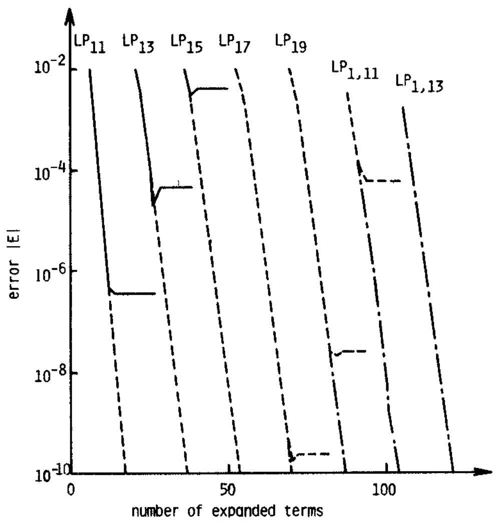

Fig. 1. Error of cutoff frequencies for uniform-core fibers computed by the PSEM, as functions of the number of terms N. Solid curves, broken curves, and dash-dotted curves show the errors for single, double, and quadruple precision computations, respectively.

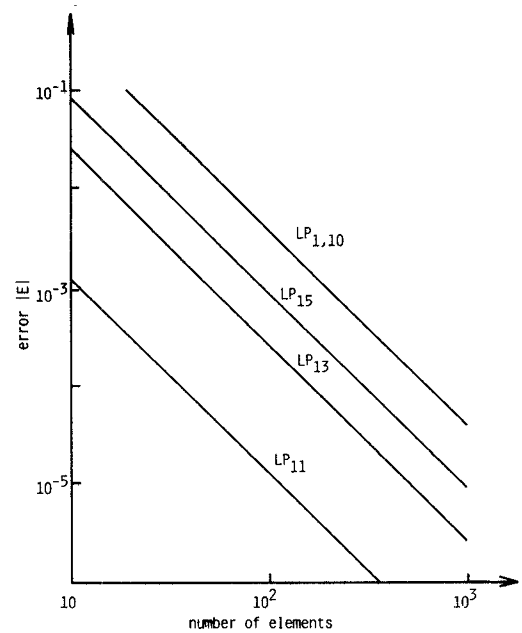

Fig. 2. Error of cutoff frequencies for uniform-core fibers computed by the FEM, as functions of the number of elements M.

Fig. 1 shows the error defined as

$$E=\frac{v_{c,\,\text{computed}}-v_{c,\,\text{exact}}}{v_{c,\,\text{exact}}} \tag{24}$$

obtained with the PSEM, as functions of the number of the expansion terms N. Fig. 2 shows the error E of the FEM as functions of the number of elements M in the finite-element analysis.

These figures suggest that the PSEM is more accurate than the FEM, provided that v is relatively small and/or

TABLE I
NORMALIZED CUTOFF FREQUENCIES OF LP_{ml} MODES FOR α-POWER PROFILES ($\rho=1$) COMPUTED BY USING THE PSEM (REPRODUCED DIRECTLY FROM COMPUTER OUTPUT. MODE NUMBERS m, l ARE TYPED AS M, L)

ALPHA = 1

	M = 0	M = 1	M = 2	M = 3	M = 4
L = 1	0.000000	4.381552	7.218053	9.918859	12.569812
L = 2	5.948312	8.933073	11.714840	14.414960	17.073726
L = 3	10.773218	13.575572	16.301533	18.981256	21.632252
L = 4	15.535689	18.247972	20.930571	23.587745	26.226232
L = 5	20.277313	22.934087	25.583111	28.219431	30.844425
L = 6	25.009182	27.627715	30.250148	32.868007	35.479889
L = 7	29.735625	32.325942	34.926809	37.528495	40.128122
L = 8	34.458707	37.027206	39.610216	42.197722	44.786075
L = 9	39.179548	41.730590	44.298550	46.873568	49.451613
L =10	43.898814	46.435519	48.990603	51.554558	54.123189

	M = 5	M = 6	M = 7	M = 8	M = 9
L = 1	15.197459	17.812575	20.421423	23.025945	25.628033
L = 2	19.709153	22.330576	24.943276	27.550414	30.153954
L = 3	24.264756	26.885041	29.497118	32.103625	34.706342
L = 4	28.851316	31.466823	34.075488	36.679279	39.279614
L = 5	33.460362	36.069329	38.673037	41.272834	43.869767
L = 6	38.086220	40.687894	43.285842	45.880906	48.473796
L = 7	42.724979	45.319145	47.910979	50.500916	53.089385
L = 8	47.373859	49.960595	52.546222	55.130863	57.714716
L = 9	52.030832	54.610369	57.189855	59.769165	62.348452
L =10	56.694386	59.267010	61.840825	64.411383	67.028140

(a)

ALPHA = 2

	M = 0	M = 1	M = 2	M = 3	M = 4
L = 1	0.000000	3.518050	5.743923	7.847594	9.904203
L = 2	5.067506	7.451446	9.645060	11.759842	13.833188
L = 3	9.157606	11.423744	13.590328	15.702133	17.780248
L = 4	13.197225	15.408067	17.554802	19.660836	21.739339
L = 5	17.220229	19.397792	21.529546	23.629565	25.706695
L = 6	21.235517	23.390452	25.510500	27.604910	29.679959
L = 7	25.246531	27.384901	29.495527	31.584878	33.657593
L = 8	29.254906	31.380530	33.483388	35.568216	37.638559
L = 9	33.261524	35.376982	37.473309	39.554097	41.622126
L =10	37.266908	39.374036	41.464782	43.541950	45.607769

	M = 5	M = 6	M = 7	M = 8	M = 9
L = 1	11.937778	13.958687	15.972134	17.980980	19.986899
L = 2	15.882132	17.915717	19.939211	21.955873	23.967812
L = 3	19.836252	21.877116	23.907310	25.929824	27.946725
L = 4	23.798431	25.843439	27.878016	29.904747	31.925509
L = 5	27.766848	29.814148	31.851566	33.881301	35.905008
L = 6	31.740089	33.788551	35.827781	37.859640	39.885574
L = 7	35.717108	37.766029	39.806371	41.839717	43.867326
L = 8	39.697134	41.746066	43.787038	45.821404	47.850265
L = 9	43.679590	45.728244	47.769510	49.804555	51.834340
L =10	47.664039	49.712229	51.753553	53.789020	55.819476

(b)

ALPHA = 4

	M = 0	M = 1	M = 2	M = 3	M = 4
L = 1	0.000000	2.999553	4.349936	6.580971	8.263079
L = 2	4.555590	6.609376	8.464058	10.231664	11.951132
L = 3	8.226613	10.202567	12.058915	13.846820	15.590939
L = 4	11.855771	13.795271	15.650208	17.449592	19.210069
L = 5	15.470632	17.388235	19.240901	21.047174	22.818915
L = 6	19.078758	20.981466	22.331718	24.642327	26.422112
L = 7	22.683119	24.574905	26.422839	28.236268	30.021978
L = 8	26.285139	28.168503	30.014287	31.829588	33.619775
L = 9	29.885592	31.762224	33.606037	35.422593	37.216237
L =10	33.484940	35.356041	37.198055	39.015451	40.811810

	M = 5	M = 6	M = 7	M = 8	M = 9
L = 1	9.919787	11.561690	13.194344	14.820944	16.443449
L = 2	13.640769	15.310610	16.966578	18.612914	20.251867
L = 3	17.305095	18.997798	20.674632	22.339436	23.994945
L = 4	20.942088	22.652626	24.346560	26.027414	27.697811
L = 5	24.564169	26.288640	27.996502	29.690900	31.374249
L = 6	28.177403	29.912895	31.632152	33.337945	35.032461
L = 7	31.785053	33.529395	35.258060	36.973481	38.677623
L = 8	35.389006	37.140562	38.877075	40.600686	42.313156
L = 9	38.990421	40.747936	42.491064	44.221694	45.941406
L =10	42.590240	44.352537	46.101294	47.837985	49.564030

(c)

ALPHA =10

	M = 0	M = 1	M = 2	M = 3	M = 4
L = 1	0.000000	2.649259	4.242970	5.713952	7.130066
L = 2	4.174340	6.026765	7.675864	9.228729	10.724173
L = 3	7.591270	9.378183	11.042524	12.632376	14.171512
L = 4	10.958339	12.718355	14.390161	16.001372	17.568038
L = 5	14.309667	16.054404	17.730210	19.355070	20.940528
L = 6	17.653809	19.388406	21.066491	22.700703	24.299729
L = 7	20.993947	22.721218	24.400646	26.041594	27.650827
L = 8	24.331596	26.053267	27.733492	29.379478	30.996638
L = 9	27.667573	29.384796	31.065481	32.715347	34.338818
L =10	31.002366	32.715950	34.396862	36.049806	37.678405

	M = 5	M = 6	M = 7	M = 8	M = 9
L = 1	8.515250	9.880903	11.233353	12.576461	13.912734
L = 2	12.181310	13.611131	15.020595	16.414397	17.795860
L = 3	15.673698	17.147726	18.599600	20.033620	21.452985
L = 4	19.100437	20.605606	22.088605	23.553210	25.002323
L = 5	22.494501	24.022693	25.529368	27.017809	28.490604
L = 6	25.869832	27.415705	28.940963	30.448461	31.940500
L = 7	29.233415	30.793270	32.333486	33.856557	35.364526
L = 8	32.569152	34.160326	35.712832	37.248857	38.770223
L = 9	35.939396	37.519910	39.062683	40.629648	42.162433
L =10	39.265653	40.873995	42.445467	44.001788	45.544419

(d)

the numerical error is reduced by increasing N and the significant digit in the computation. However, generally the PSEM is more time consuming. As seen in Fig. 1, to achieve satisfactory accuracy for higher modes ($l>9$), the PSEM requires the "quadruple precision" computation which means to deal with about 33 significant digits. In the FEM, when $M<1000$, double precision computation (17 significant digits) is usually enough.

In the PSEM the computing time is proportional to N. In the FEM it is approximately proportional to M. (When we compute the determinant of an $M\times M$ matrix, the time required is usually proportional to M^2. However, in the present case most of the off-diagonal elements are zero.)

B. Normalized Cutoff Frequencies for α-Power Profiles

The normalized cutoff frequencies of LP_{ml} modes for α-power profiles ($\alpha=1$, 2, 4, and 10, $\rho=1$) computed by PSEM are shown in Table I. It has been confirmed that the relative difference between the v_c values obtained by the FEM and PSEM ($(v_{c\,\mathrm{FEM}}-v_{c\,\mathrm{PSEM}})/v_{c\,\mathrm{PSEM}}$) is below 0.005 percent (5×10^{-5}) except for only two cases ($LP_{8,10}$ and $LP_{9,10}$ modes, both for $\alpha=1$), and below 0.06 percent over the entire table.

C. Dispersion and Delay-Time Characteristics

Figs. 3–8 show the dispersion and delay time characteristics (curve-plotter output) of LP_{ml} modes for six α-power profiles: $(\alpha,\rho)=(1.1)$, $(2,1)$, $(4,1)$, $(10,1)$, $(2,1.5)$, and $(2,2)$. The upper and lower figures for each case show the dispersion and delay-time characteristics, respectively. The abscissa gives the normalized frequency v. The ordinates of the upper and lower figures give a conventionally used parameter representing β

$$X=(k^2n_1^2-\beta^2)/(k^2n_1^2-k^2n_2^2) \tag{25}$$

and the normalized delay time

$$T=\frac{ct}{N_1}-1 \tag{26}$$

where c denotes the light velocity, t the delay time per unit distance (19), and N_1 is the group index of the material at the center of the core (it is assumed to be equal to n_1 in the present analysis).

The FEM ($M=100$) has been used to obtain these graphs. However, computations have also been performed by using the PSEM. It has been confirmed that the relative error in the horizontal direction (i.e., $(v_{\mathrm{FEM}}-v_{\mathrm{PSEM}})/v_{\mathrm{PSEM}}$ for prescribed X or T) is less than 10^{-2} over the entire graphs, and much lower than 10^{-2} at most parts.

Incidentally, comparison of Figs. 3–6 suggests the well-known superiority of the quadratic profile ($\alpha=2$) in reducing the intermodal delay difference; the best "bunching" of $T-v$ curves is found in Fig. 4. Comparison of

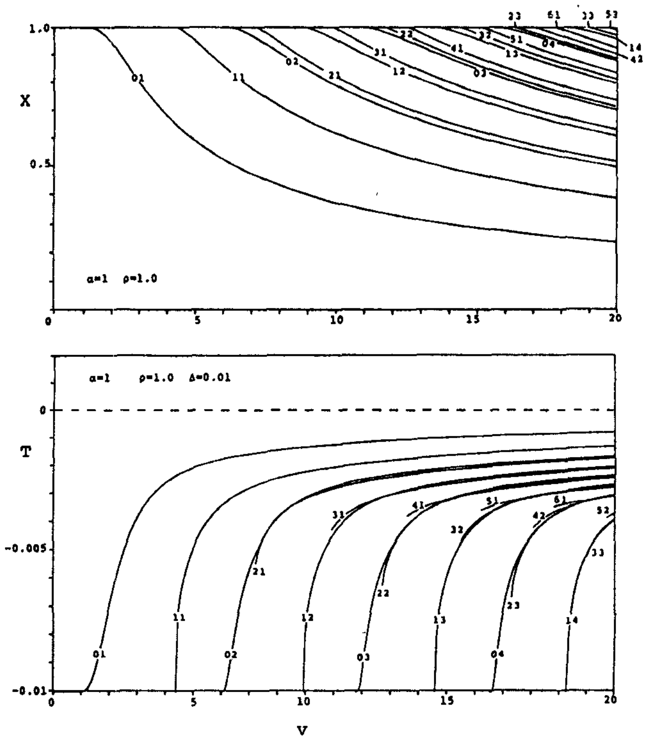

Fig. 3. Propagation characteristics for the case $\alpha = 1$, $\rho = 1$, computed by using the FEM. The numerals for each curve show the LP mode numbers m and l, e.g., 34 denotes the LP_{34} mode.

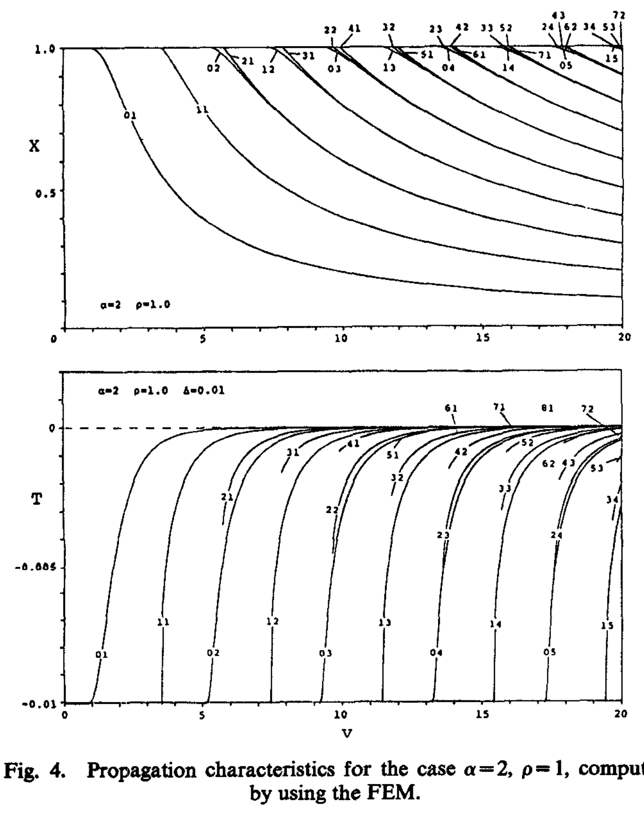

Fig. 4. Propagation characteristics for the case $\alpha = 2$, $\rho = 1$, computed by using the FEM.

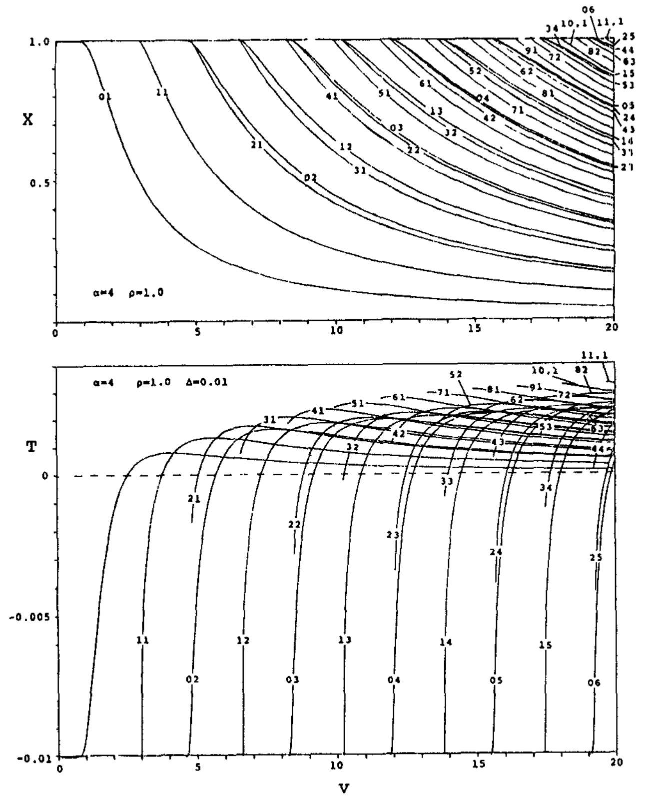

Fig. 5. Propagation characteristics for the case $\alpha = 4$, $\rho = 1$, computed by using the FEM.

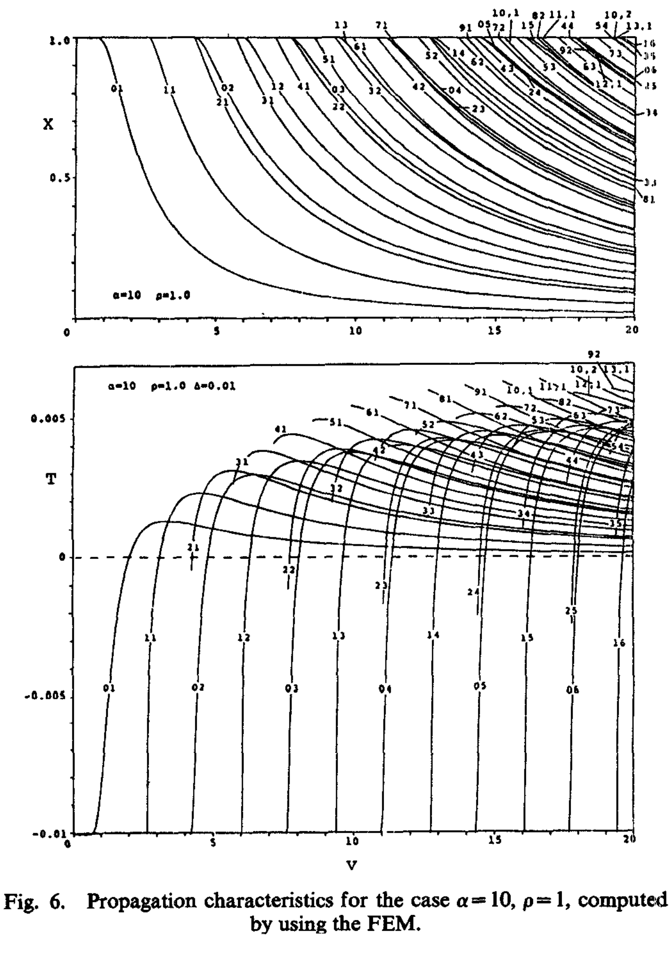

Fig. 6. Propagation characteristics for the case $\alpha = 10$, $\rho = 1$, computed by using the FEM.

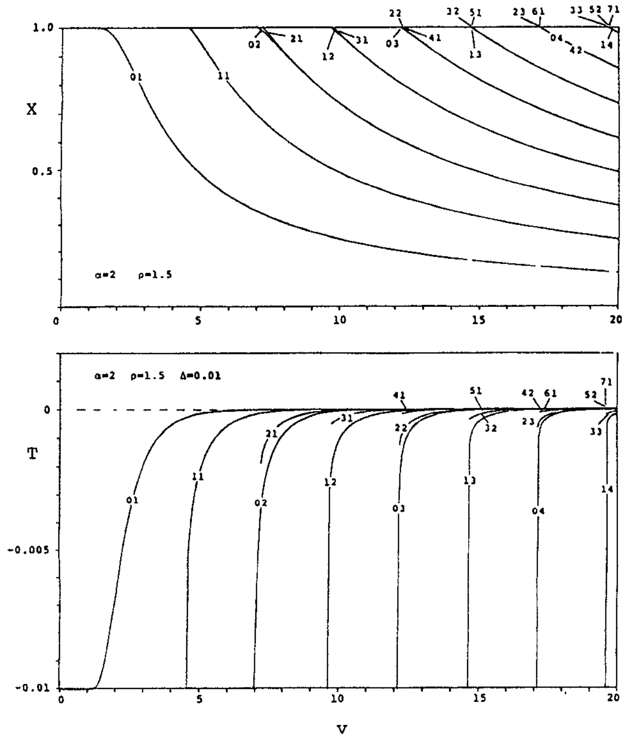

Fig. 7. Propagation characteristics for the case $\alpha=2$, $\rho=1.5$ (a quadratic profile with valley) computed by using the FEM.

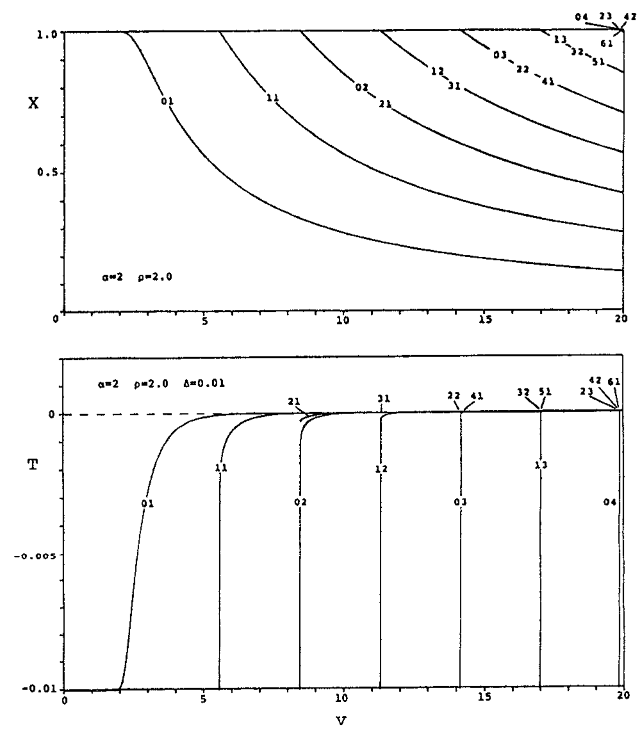

Fig. 8. Propagation characteristics for the case $\alpha=2$, $\rho=2$ (a quadratic profile with valley) computed by using the FEM.

Figs. 4, 7, and 8 suggests that the delay difference can further be reduced dramatically by making an index valley at the core-cladding boundary ($\rho=1.5$ or $\rho=2$), as proposed by Okamoto and Okoshi [10][1], [12].

VI. Conclusion

High-accuracy data of normalized cutoff frequencies, propagation constants and delay time of LP_{ml} modes for α-power graded-core fibers have been presented. Computations have been performed by two entirely different methods, and the results are compared to assure accuracy. The presented data will be useful for estimating the errors of other methods of analysis.

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[1]The dispersion curves shown in [10] are a little different from Figs. 3–8 of the present paper. This is because the closed-form dispersion formula, which was used to compute the dispersion curves, is an approximate one. This fact was overlooked in [10]. The same formula is newly derived in a correct manner but as an approximation in [11].